

On Cryptography and Distribution Verification

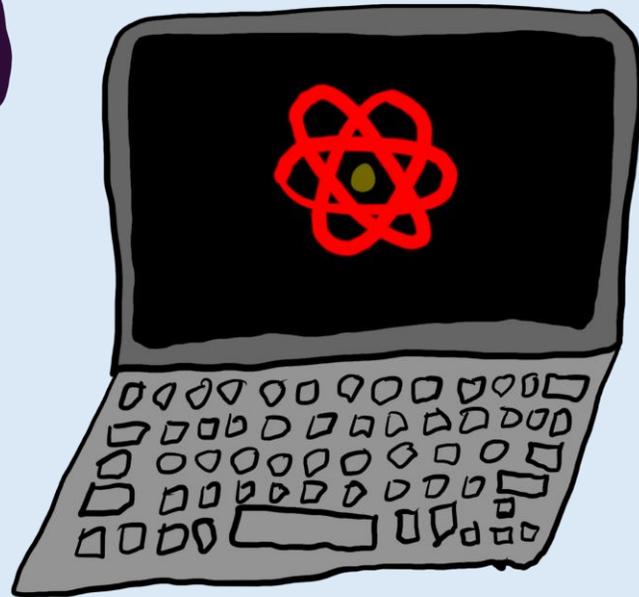
with Applications to Quantum Advantage

Bruno Cavalari, **Eli Goldin**, Matthew Gray, Taiga Hiroka, Tomoyuki Morimae

Quantum Advantage



Hey everyone,
check out my
quantum computer!



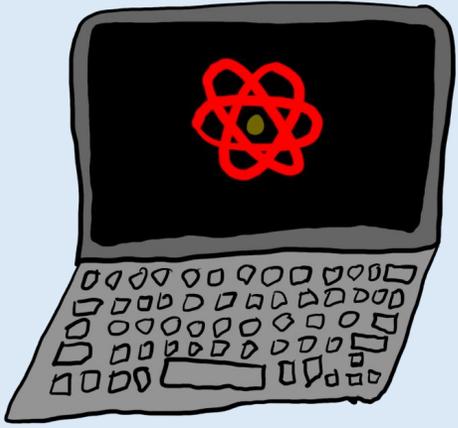
Quantum Advantage



Ha ha!
Fooled you!



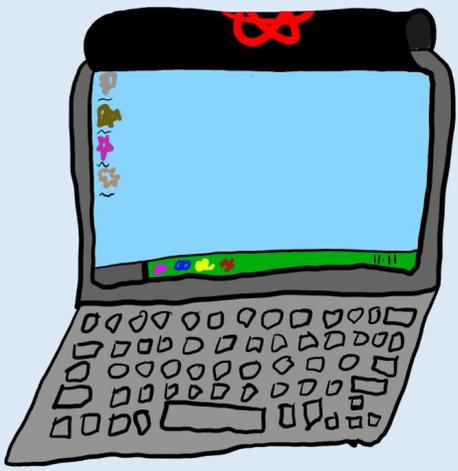
Proof of Quantumness



- yep,
quantum

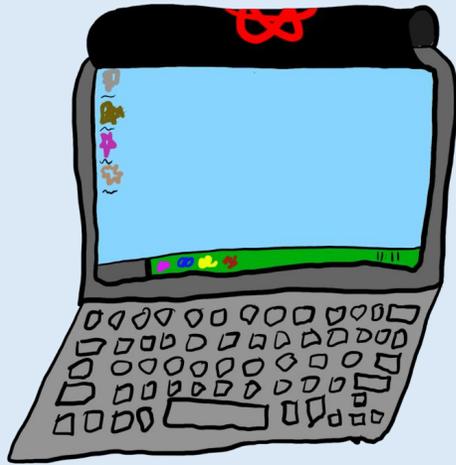
Proof of Quantumness

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Proof of Quantumness

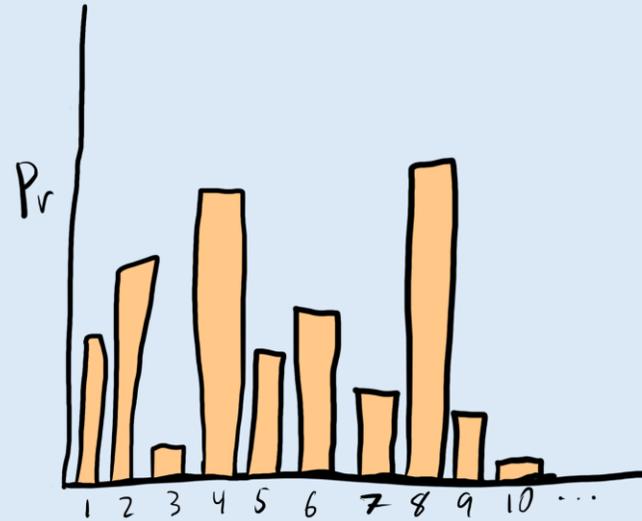
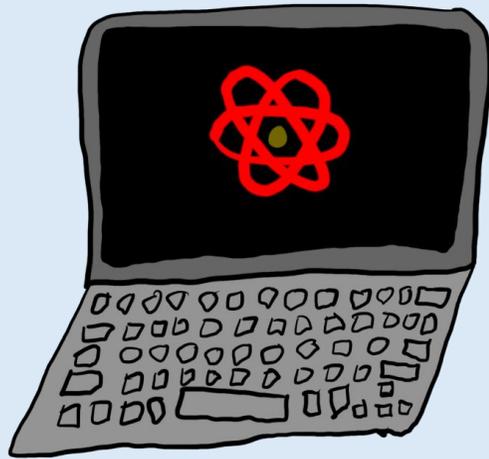
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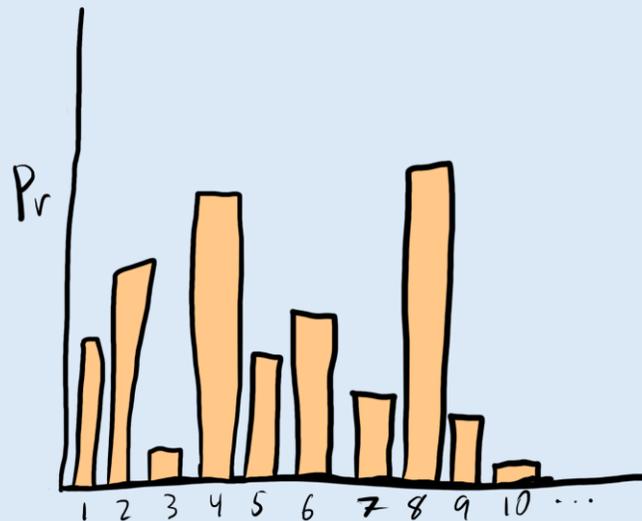
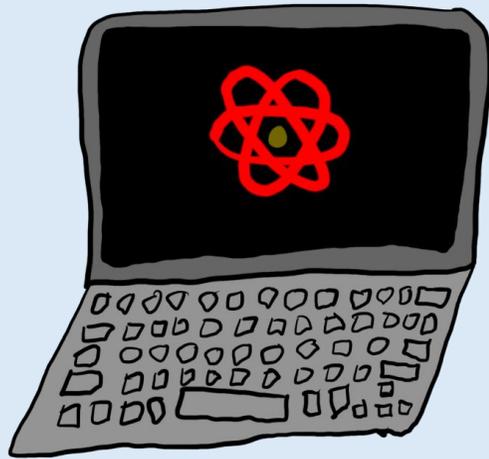
EXPENSIVE

[BCM+18, BKVV20, KMLVY21, MY22, YZ22, KLVY23, NZ23, BK

Quantum Advantage Samplers

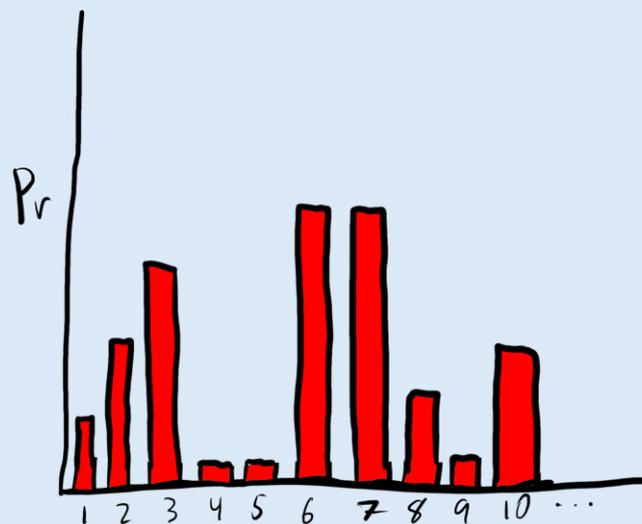
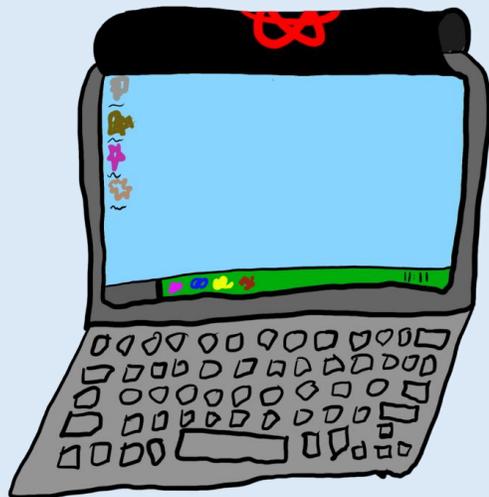


Quantum Advantage Samplers



statistically
far

V

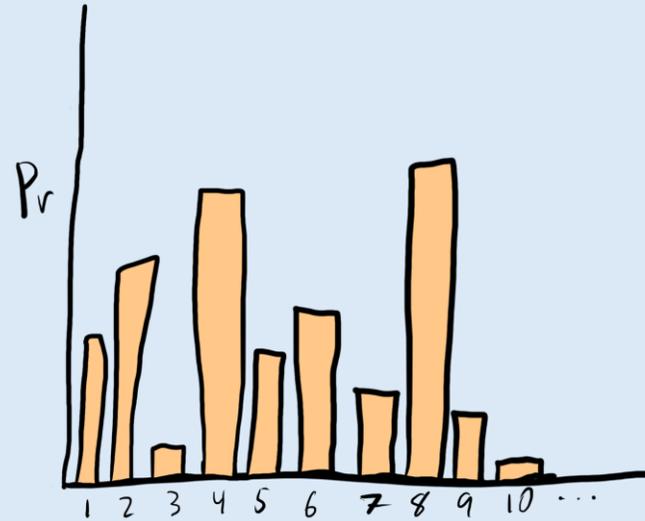
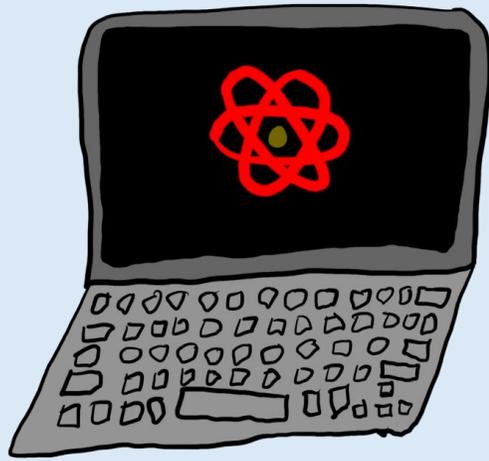


Quantum Advantage Samplers

Examples:

- Random circuit sampling [BFNV19]
- IQP circuits [BJS11]
- Boson sampling [AA11]

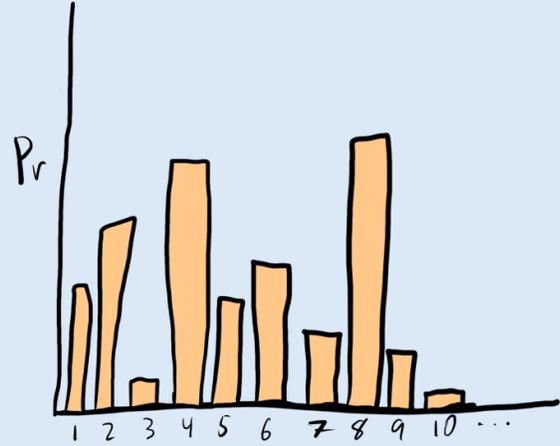
Quantum Advantage Samplers



No verifier :(

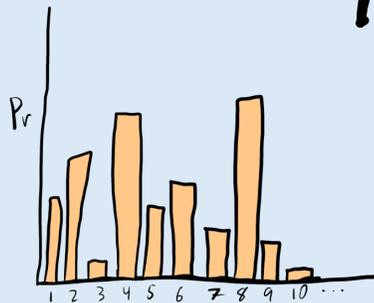
Distribution Verification

Input:

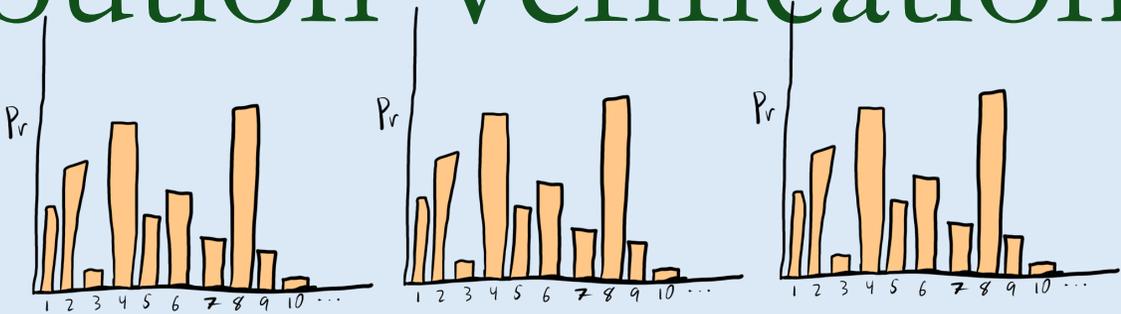
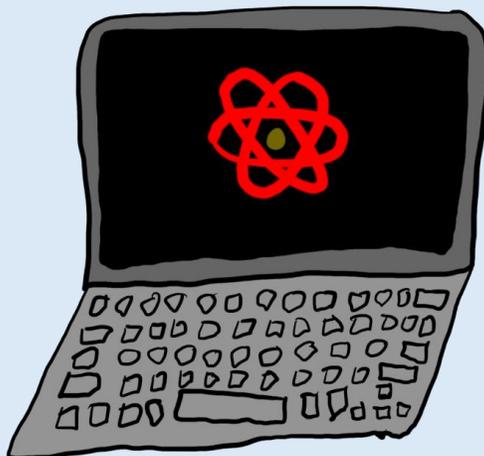


x_1, \dots, x_t

Goal: Test if x_1, \dots, x_t came from

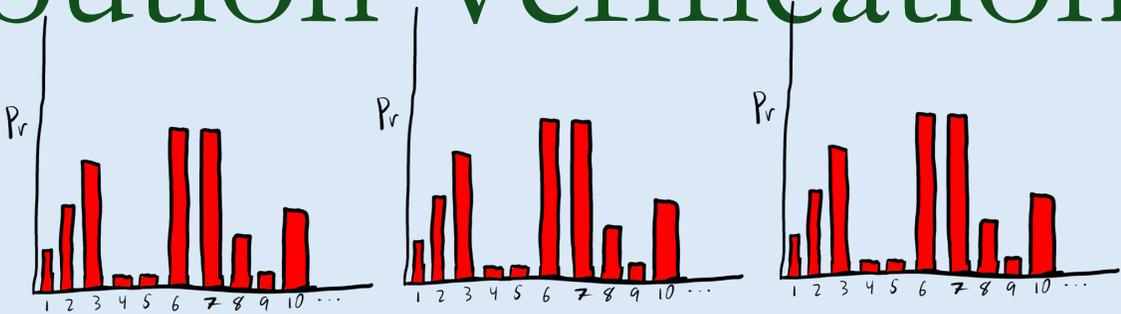
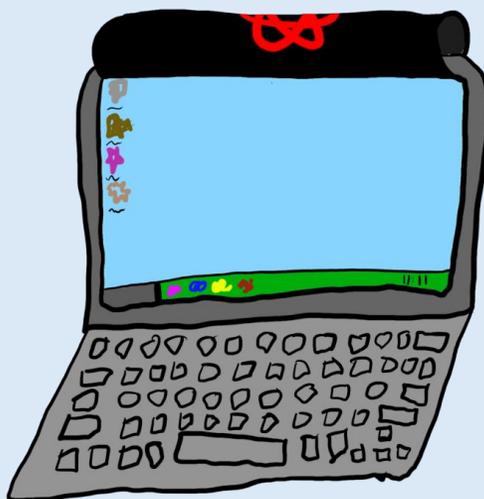


Distribution Verification + QAS



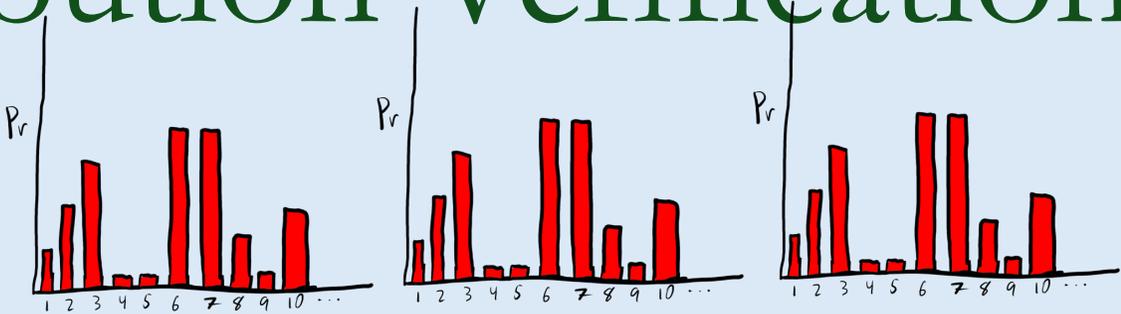
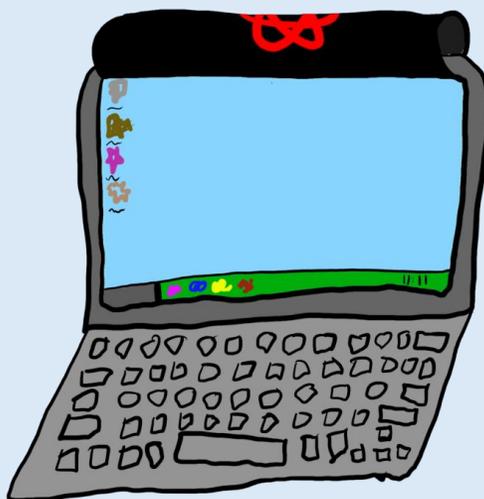
looks good to me

Distribution Verification + QAS



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Distribution Verification + QAS



IMPOSTER

* uniform adversaries

Distribution Verification

Well studied in setting of
property testing (identity test)

[BFR⁺00, BFR⁺01, DKN15, Pan 08, VV17, ...]

Identity Testing

Identity test for $D: \text{Ver s.t.}$

- (Correctness): $\Pr_{D^+ \rightarrow x_1, \dots, x_t} [\text{Ver}(x_1, \dots, x_t) \rightarrow 1] \geq \text{high}$

Identity Testing

Identity test for \mathcal{D} : Ver s.t.

- (Correctness): $\Pr_{\mathcal{D}^t \rightarrow x_1, \dots, x_t} [\text{Ver}(x_1, \dots, x_t) \rightarrow 1] \geq \text{high}$

- (Security): \forall inefficient $\mathcal{G}, \Delta(\mathcal{G}, \mathcal{D}) \geq \epsilon$
 $\Pr_{\mathcal{G}^t \rightarrow x'_1, \dots, x'_t} [\text{Ver}(x'_1, \dots, x'_t) \rightarrow 1] \leq \text{low}$

Identity Testing

COOL

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Identity Testing

[BFF⁺01, Pan08, VV17]:

D over $\{0,1\}^n$

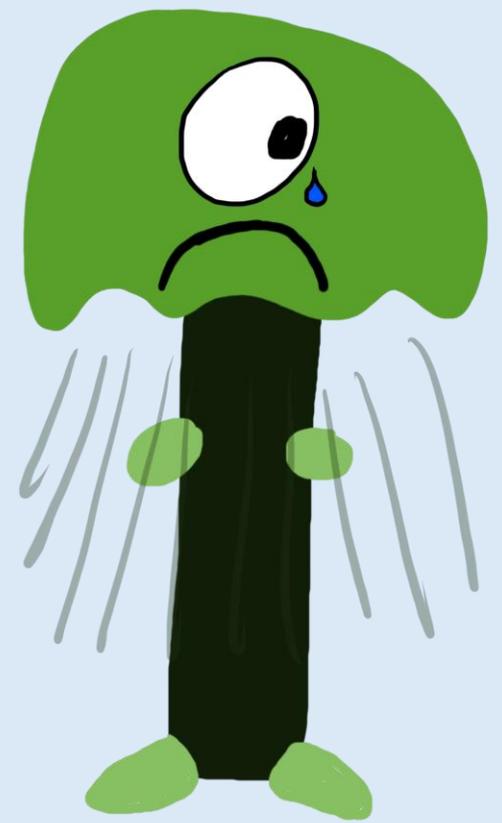
optimal sample complexity: $\Omega(2^{n/2})$

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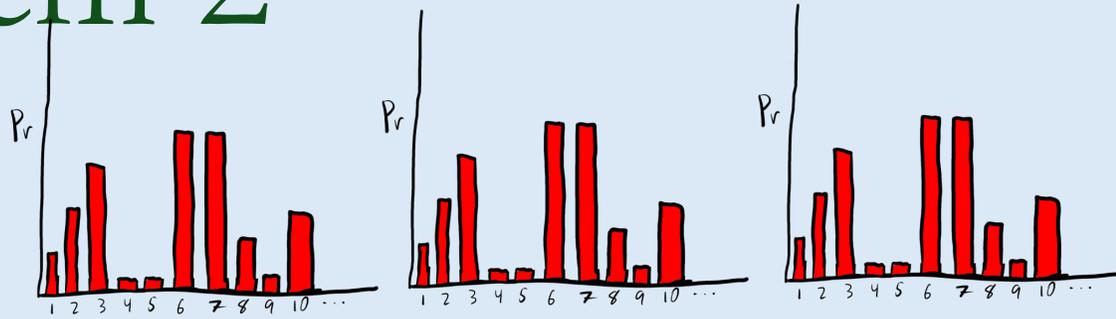
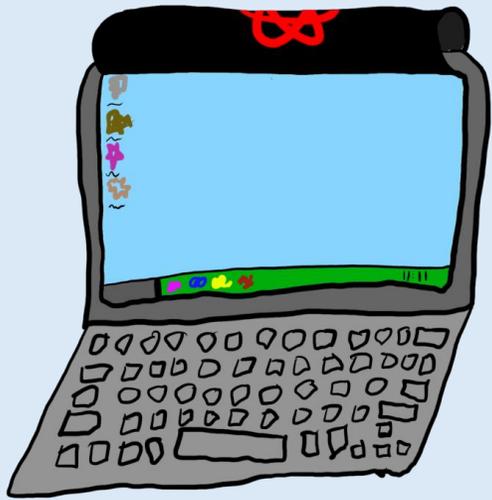
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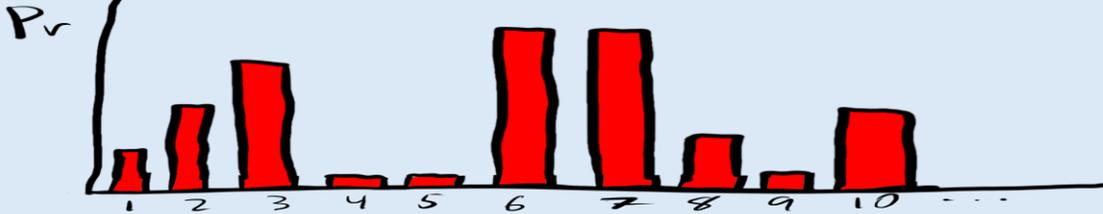
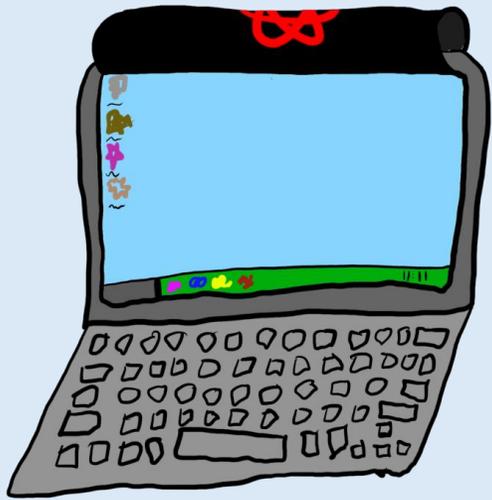
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Problem 2



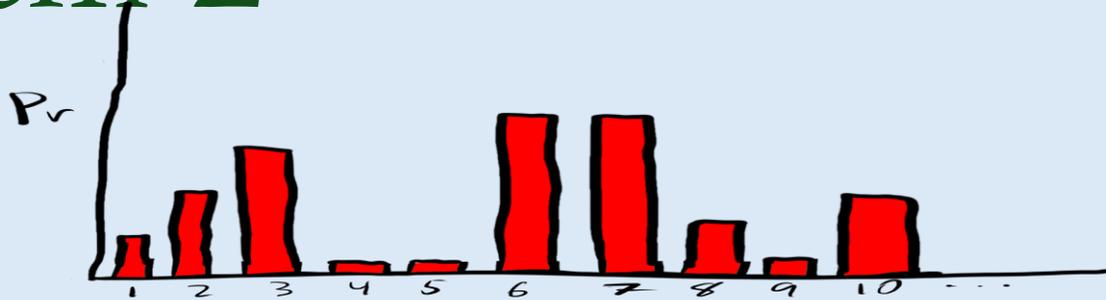
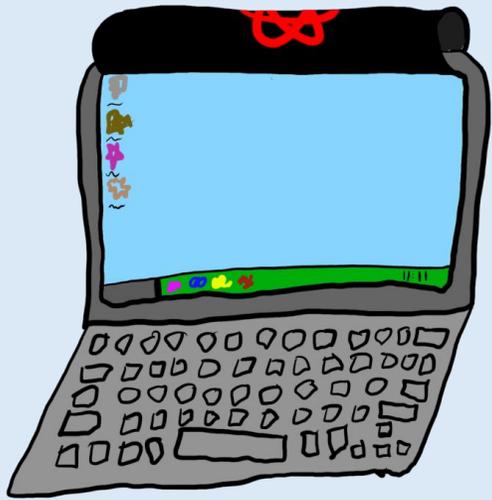
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Problem 2



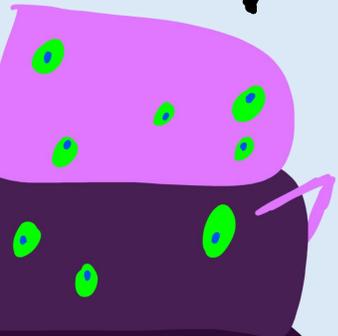
hmm, looks quantum
to me...

Problem 2



FOOLED YOU
AGAIN!

hmm, looks quantum
to me...



Distribution Verification

D efficiently verifiable if \exists eff. Ver st.

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gets ϵ to runtime of G

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Main Question

When is a (classical/quantum)
distribution efficiently
verifiable?

Our Results

Classical Setting

- if OWFs exist, then every sufficiently random samplable dist. is **NOT** efficiently verifiable.

Classical Setting $H_\infty \geq \omega(\log n)$

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- if OWFs do not exist, then every samplable dist. **IS** efficiently verifiable. !!

Quantum Setting

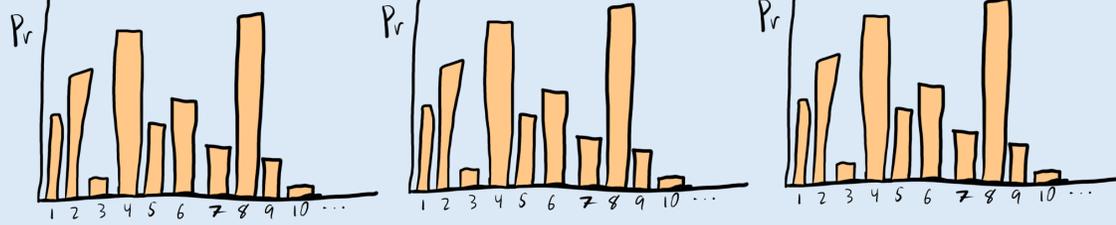
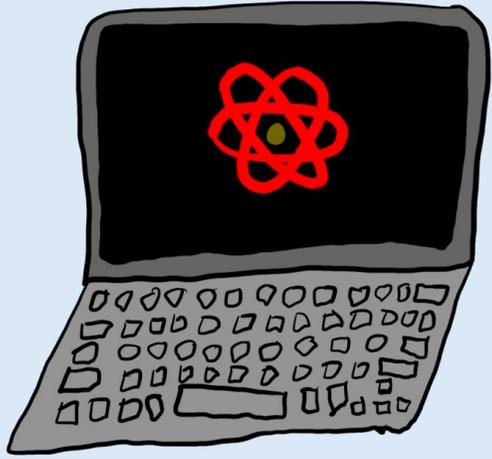
- If one-way puzzles do not exist, then every quantum advantage sampler is efficiently verifiable with security against PPT machines

Quantum Setting

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Note: Ver is quantum

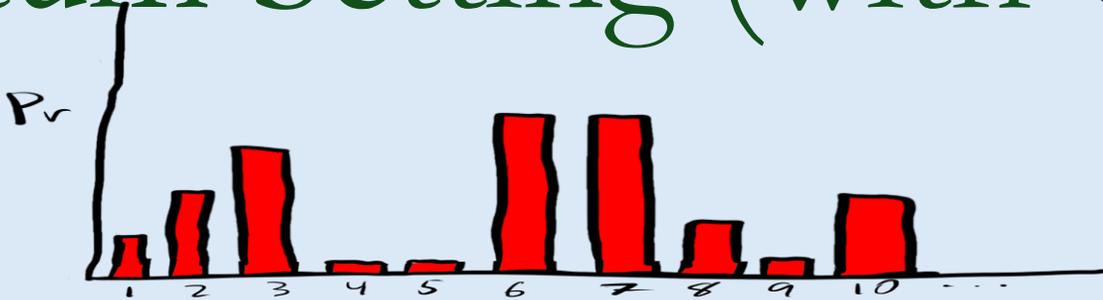
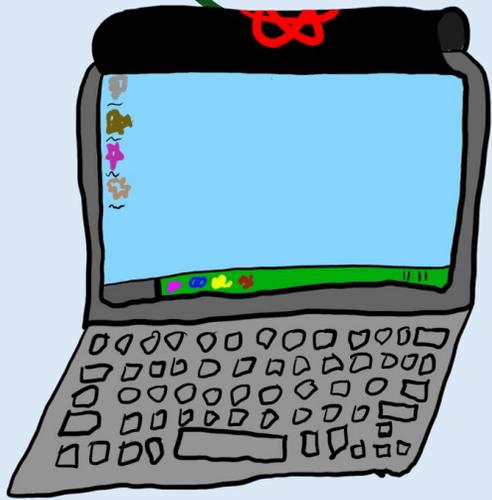
Quantum Setting (with QAS)



GOOD ENOUGH

looks good to me

Quantum Setting (with QAS)



GOOD ENOUGH

Ha! Nice try.

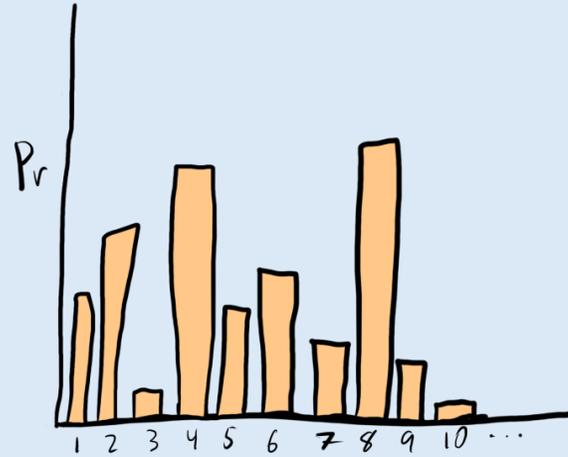
Techniques

Classical Setting $H_\infty \geq \omega(\log n)$

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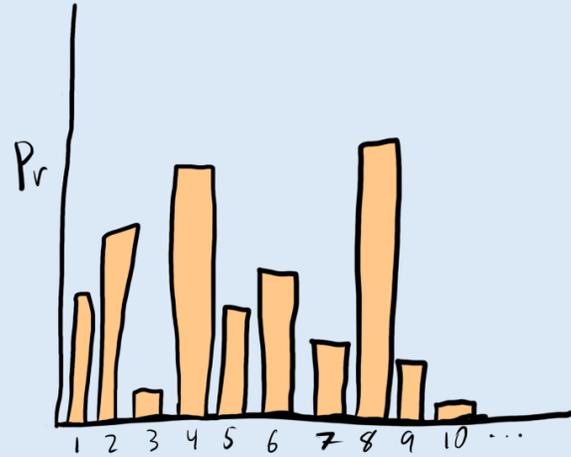
OWF Counterexample

$D(r)$ →



OWF Counterexample

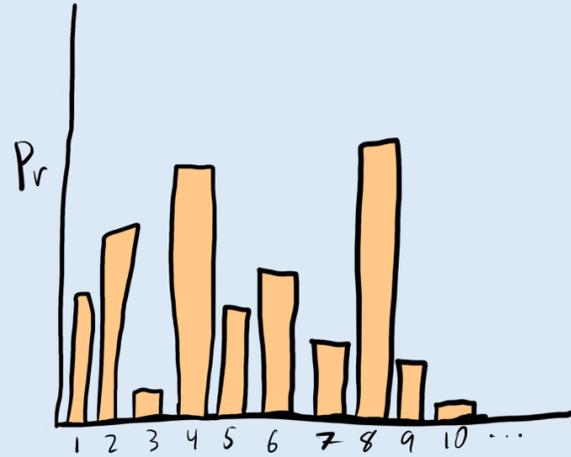
$D(G) \rightarrow$



, G a PRG

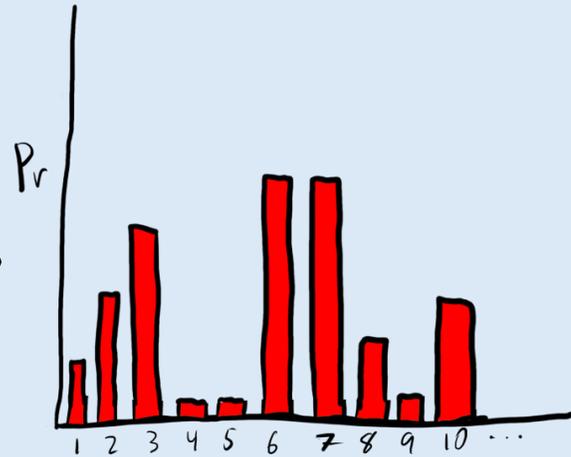
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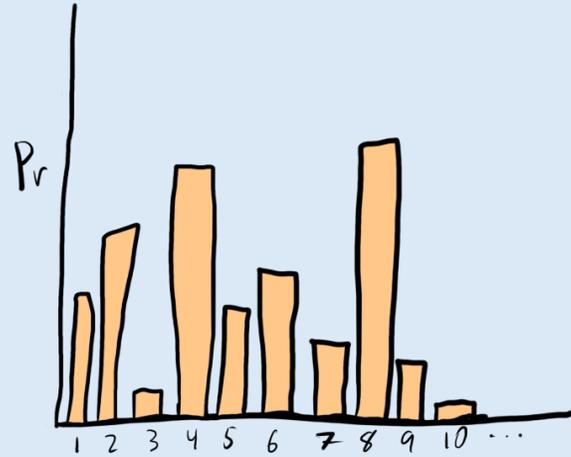
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$D(G(r)) \rightarrow$



OWF Counterexample

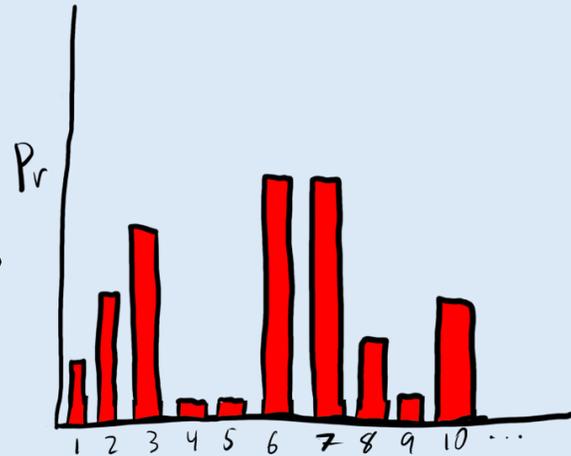
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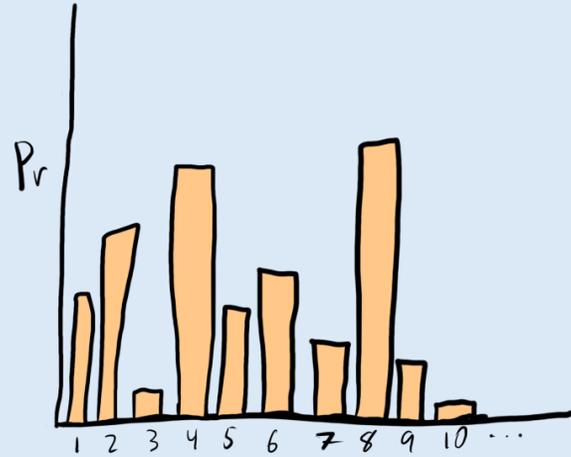
Statistically far!

$D(G(r)) \rightarrow$



OWF Counterexample

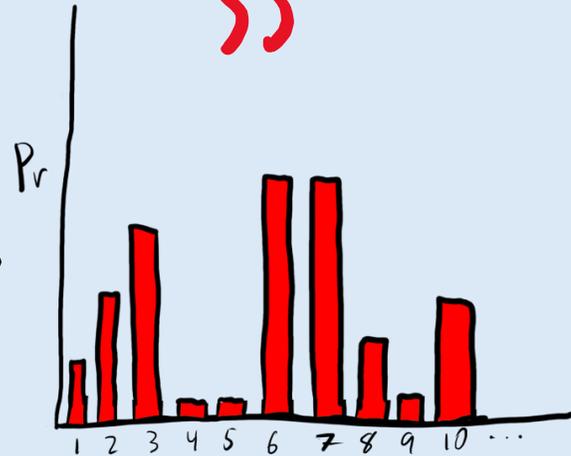
$D(r) \rightarrow$



G a PRG

Statistically far!
Indistinguishable!

$D(G(r)) \rightarrow$



SS

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Classical Setting

- if OWFs do not exist, then every samplable dist. **IS** efficiently verifiable.

Classical Setting

Key Tool: Kolmogorov Complexity

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Kolmogorov Complexity

$K(x)$ = length of the shortest Turing machine
outputting x .

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$\times 1000$

$K(1010 \dots 10)$

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$$K(\overbrace{1010\dots 10}^{\times 1000}) = | \text{"output 10 1000 times"} | = 20$$

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$$K(\underbrace{10110001010011\dots}_{1000 \text{ random characters}})$$

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$$K(\underbrace{10110001010011\dots}_{1000 \text{ random characters}}) \approx 1000$$

Kolmogorov Complexity

Fact: \mathcal{D} samplable dist.,
w.h.p. $\mathcal{D} \rightarrow x$, $K(x) \approx \log \overline{\Pr[\mathcal{D} \rightarrow x]}$

Kolmogorov Complexity

Fact: \mathcal{D} samplable dist.,
w.h.p. $\mathcal{D} \rightarrow x$, $K(x) \approx \log \frac{1}{\Pr[\mathcal{D} \rightarrow x]}$

[LOZ1] Coding Theorem

$\forall x$,

$$K(x) \leq |\mathcal{D}| + \log \left(\frac{1}{\Pr[\mathcal{D} \rightarrow x]} \right) + c$$

Kolmogorov Complexity

Fact: \mathcal{D} samplable dist.,

$$\text{w.h.p. } \mathcal{D} \rightarrow x, \quad K(x) \approx \log \overline{\Pr[\mathcal{D} \rightarrow x]}$$

[LZ77] Coding Theorem ; [LV93] Incompressibility

$\forall x,$; w.h.p. over $\mathcal{D} \rightarrow x,$

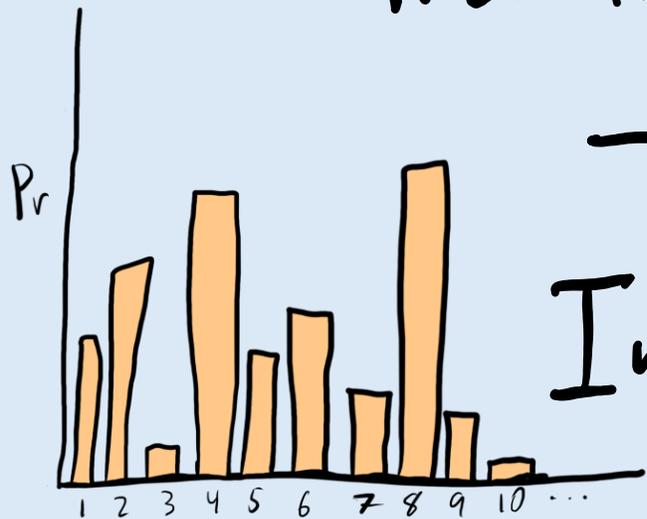
$$K(x) \leq |\mathcal{D}| + \log \left(\frac{1}{\Pr[\mathcal{D} \rightarrow x]} \right) + c$$

$$K(x) \geq \log \left(\frac{1}{\Pr[\mathcal{D} \rightarrow x]} \right) - c$$

Inefficient Verifier [Aaronson14]

$\text{Ver}(x_1, \dots, x_t)$: output 1 \iff

$$\log \frac{1}{\Pr[D \rightarrow x_1] \dots \Pr[D \rightarrow x_t]} \leq K(x_1, \dots, x_t) + c$$



$\rightarrow x_1, \dots, x_t$

Incompressibility: w.h.p. ✓

Inefficient Verifier [Aaronson14]

$\text{Ver}(x_1, \dots, x_t)$: output 1 \Leftrightarrow

$$\log \frac{1}{\Pr[D \rightarrow x_1] \cdots \Pr[D \rightarrow x_t]} \leq K(x_1, \dots, x_t) + c$$

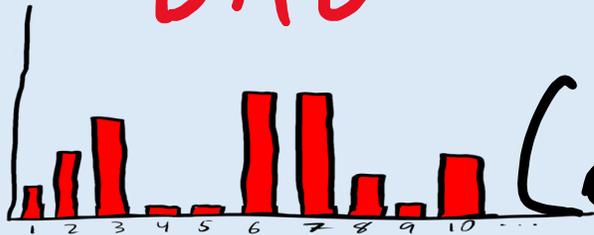
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BAD

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Coding thm: $K(x_1, \dots, x_t) \leq \log \frac{1}{\Pr[D' \rightarrow (x_1, \dots, x_t)]} + c$

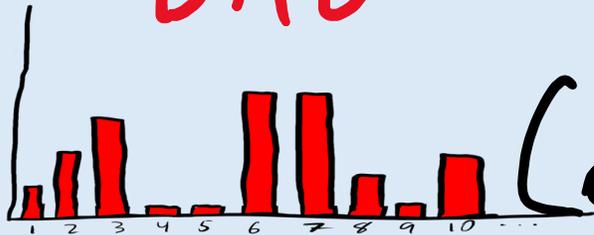
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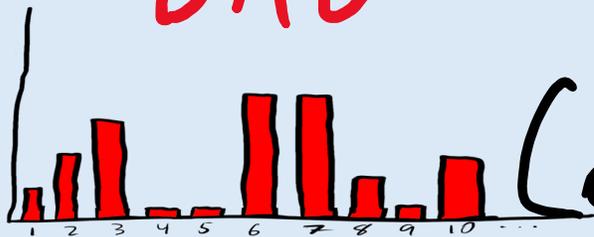
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Efficient Verifier

Idea: if OWFs don't exist, can estimate

$$\log \frac{1}{\Pr[D \rightarrow x_1] \cdots \Pr[D \rightarrow x_2]} \quad (\text{prob. estimation})$$

[IL89]

Probability Estimation

[IL89]: For all dist. D , there exists

Est s.t. w.h.p. $D \rightarrow x$,

$$\text{Est}(x) \approx -\log \Pr[D \rightarrow x]$$

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(l-sided error): $\forall x, \text{Est}(x) \geq -\log \Pr[D \rightarrow x]$

Efficient Verifier

Idea: if OWFs don't exist, can estimate

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Problem: $K(x_1, \dots, x_t)$ uncomputable

Efficient Verifier

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[IL89]

Problem: $K(x_1, \dots, x_t)$ uncomputable

Solution: Use different type of K -complexity

$$uK^t(x)$$

$$uK^t(x) = -\log \text{Pr}[\text{random TM outputs } x]$$

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$$= -\log_{\Pi \leftarrow \{0,1\}^t} \Pr[U\text{TM}^t(\Pi) \rightarrow x]$$

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Coding theorem \rightarrow incompressibility still hold!

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Coding theorem & incompressibility still hold!

$$\text{Ver}(x_1, \dots, x_t) \rightarrow \log \frac{1}{\Pr[D \rightarrow x_1] \cdots \Pr[D \rightarrow x_t]} \leq uK^t(x) + c$$

works!

Estimating uK^t

Thm [HN23]: If OWFs do not exist, then
there exists Est such that for ALL samplable \mathcal{D} ,
w.h.p. $\mathcal{D} \rightarrow x$, $\text{Est}(x) \approx uK^t(x)$

Final Verifier

$$\text{Ver}(x_1, \dots, x_t) \rightarrow 1 \Leftrightarrow \log \underbrace{\Pr[D \rightarrow x_1] \cdots \Pr[D \rightarrow x_t]}_{\substack{\text{replace w/ approximations} \\ \text{you get via no OWFs}}} \leq \underbrace{uK^+(x)}_{\text{wavy line}} + c$$

Final Verifier

$$\text{Ver}(x_1, \dots, x_t) \rightarrow 1 \Leftrightarrow \log \underbrace{\Pr[D \rightarrow x_1] \cdots \Pr[D \rightarrow x_t]}_1 \leq \underbrace{uK^+(x)}_1 + c$$

replace w/ approximations
you get via no OWFs

D good \Rightarrow approximations right \Rightarrow passes Ver

Final Verifier

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G far

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G far \Rightarrow right, lower bounded

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replace w/ approximations
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\hookrightarrow far \Rightarrow right, lower bounded \Rightarrow test fails
1-sided error critical!

Classical Setting



- if OWFs do not exist, then every samplable dist. **IS** efficiently verifiable.

Quantum Setting

- If one-way puzzles do not exist, then every quantum advantage sampler is efficiently verifiable with security against PPT machines

Quantum Setting

OWPuzz: quantum analogue of OWF

Quantum Setting

OWPuzzle: quantum analogue of OWF

quantum
t

Samp \rightarrow (k, s)

classical

Quantum Setting

OWPuzz: quantum analogue of OWF

quantum
↓

classical
↑

Samp \rightarrow (k, s)

Security: Given s , hard to sample from conditional dist. on k .

Quantum Setting

OWPuzz: quantum analogue of OWF

quantum
t

classical
s

Samp $\rightarrow (k, s)$

Security: Given s , hard to sample from conditional dist. on k .

OWF \Rightarrow OWPuzz \Rightarrow quantum commitments

Quantum Setting

OWPuzz: quantum analogue of OWF

Quantum Setting

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$$q_{\mathcal{L}}^{\text{K}^+}(x) = -\log \Pr[\text{random QTM} \rightarrow x]$$

Quantum Setting

OWPuzz: quantum analogue of OWF

$$q_{\tau} K^{\dagger}(x) = -\log \Pr[\text{random QTM} \rightarrow x]$$

No OWPuzz \Rightarrow prob. est. for quantum samplable dists
[GGH25, HM25, KT25]

Quantum Setting

OWPuzz: quantum analogue of OWF

$$q_{\mathcal{K}}^{\dagger}(x) = -\log \Pr[\text{random QTM} \rightarrow x]$$

[GGH25, HM25, KT25]

No OWPuzz \Rightarrow prob. est. for quantum samplable dists
 \Rightarrow estimator for $q_{\mathcal{K}}^{\dagger}$

Quantum Setting

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[GGH25, HM25, KT25]

No OWPuzz \Rightarrow prob. est. for quantum samplable dists
 \Rightarrow estimator for $q_{\tau} K^{\dagger}$

Problem: no 1-sided error!

Quantum Setting

Key idea: If D is samplable quantumly
but NOT classically,

Quantum Setting

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but NOT classically,

w.h.p. $D \rightarrow x$

$$q_{\epsilon} u K^+(x) \ll u K^+(x)$$

Quantum Setting

Key idea: If D is samplable quantumly
but NOT classically,

To verify,
estimate both!

w.h.p. $D \rightarrow x$

$$q_u K^+(x) \ll u K^+(x)$$

Quantum Setting

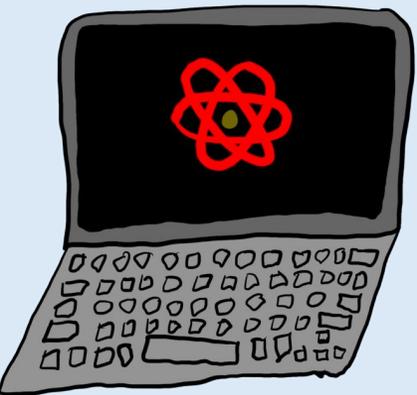
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$\text{Ver}(x_1, \dots, x_t)$: estimate $q_{uK}^{\dagger}(x_1, \dots, x_t)$
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 $\Leftrightarrow q_{uK}^{\dagger} \ll uK^{\dagger}$

Quantum Setting

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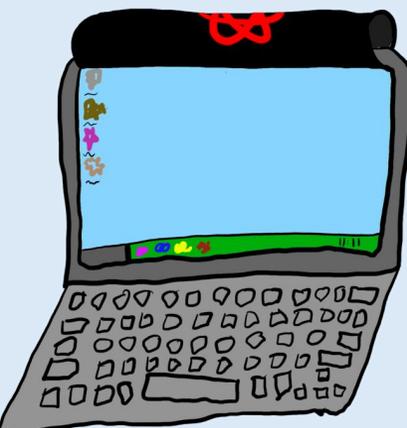
$\rightarrow q_{uK}^{\dagger} \ll uK^{\dagger} \checkmark$

Quantum Setting

D a QAS:

$\text{Ver}(x_1, \dots, x_t)$: estimate $q_{uK^t}(x_1, \dots, x_t)$
and $uK^t(x_1, \dots, x_t)$. Output 1

$$\Leftrightarrow q_{uK^t} \ll uK^t$$



$\rightarrow q_{uK^t} \approx uK^t$ **X**

Quantum Setting

- If one-way puzzles do not exist, then every quantum advantage sampler is efficiently verifiable with security against PPT machines



Open Questions

- Eff. ver. for quantum from no OWPuzz

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- Eff. ver. for quantum from no OWPuzz
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- OWPuzz \Rightarrow no dist. verifiable?
- Can we achieve weaker verifiability if OWF exist?
- Lower complexity classes?

Thank
You!

